

## Stochastic Model To Find The Effect Of Glucose-Dependent Insulinotropic Hormone Based On The Length Of Small Intestine Using Cauchy Distribution

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### ABSTRACT

The study was to evaluate the GIP response when glucose was infused into the duodenum of eight healthy males over 60 minutes on the first day into an isolated 60-cm segment of proximal small intestine. On the second day the same amount of glucose was infused to entire small intestine. GIP did not differ between days. In this paper the problem is investigated by largest discontinuity of Cauchy distribution.

**Key Words:** GIP, Endocrine k cells, Cauchy distribution

**2010 Mathematics Subject Classification:** 60G05; 60G10

### I. Introduction

Glucose was infused at 3.5kcal/min into the duodenum of eight healthy males (age 18-59 yrs) over 60 minutes on the first day into an isolated 60-cm segment of the proximal small intestine (short segment infusion). on the second day the same amount glucose was infused with access to the entire small intestine (long-segment infusion) defined in [12]. GIP is secreted from endocrine k cells [5]. GIP play an important role in glucose homeostasis, both in healthy [9] and diabetes[8]. It is important to establish how the release of peptide is controlled by luminal glucose in humans. The plasma concentrations of GIP were similar after two loads for most of that time [10]. Plasma GIP concentrations (pmol/l) were measured by radio immunoassay[13]. The sensitivity of the assay was 2 pmol/l and both the intra-assay and inter-assay coefficient of variations were 15%. In this paper the problem is investigated by valuation of the largest discontinuity in an interval proportional to the standard scale of the total Cauchy increment defined in [7]. To establish simple procedures for estimating the parameters have gone to great length to tabulate the intermediate stable symmetric distributions [3][4]. More sophisticated maximum likelihood techniques for estimating the parameters was developed [1][2]. The properties of the likelihood ratio was further investigated [11]. These intermediate stable symmetric distributions has waned in part because of their properties in a continuous time context. The continuous time stochastic process has serially independent stable increments in order for its sample path to be almost surely everywhere continuous the increment must be normal.

### Notations:

- m Scale parameter for model.
- $\Psi$  Probability of largest discontinuity
- r Equal subintervals
- H Probability of increments

### II. Stochastic Model

#### 2.1. Cauchy distribution

Let us start with the Cauchy distribution, for which  $\beta=1$ . The cumulative distribution function for the random variable with median zero and scale m is given by

$$F(x; m) = \frac{1}{2} + \frac{1}{\pi} \arctan\left(\frac{x}{m}\right) \quad (1)$$

and the probability density function [6] is given by

$$f(x;m) = \frac{1}{\pi m \left[ 1 + \left( \frac{x}{m} \right)^2 \right]} \quad (2)$$

For a Cauchy distribution, the scale is the semi interquartile range . The probability that the absolute value of such a variable will be less than  $x$  is therefore

$$H(x;m) = \frac{2}{\pi} \arctan\left(\frac{x}{m}\right) \quad (3)$$

which has probability density function

$$h(x;m) = \frac{2}{\pi m \left[ 1 + \left( \frac{x}{m} \right)^2 \right]} \quad (4)$$

Now consider an interval over which the increment in a continuous time Cauchy process has scale  $m$ , which is to say that the median absolute increment is 'm'. The scale of the sum of two Cauchy variables is the sum of their standard scales, so if we divide this interval into ' $r$ ' equal subintervals, the increment across each subinterval will have standard scale  $\frac{m}{r}$ . The probability that all of these increments will be less than  $x$  in

absolute value is  $H\left(x; \frac{m}{r}\right)^r$ .

Therefore  $\psi(x;m)$ , the probability that the largest discontinuity is less than  $x$ , is

$$\psi(x;m) = \lim_{r \rightarrow \infty} H\left(x; \frac{m}{r}\right)^r = \lim_{r \rightarrow \infty} \left( \frac{2}{\pi} \arctan \frac{rx}{m} \right)^r \quad (5)$$

Setting  $\mu = \frac{1}{r}$ , and using L' Hôpital's rule, we have

$$\begin{aligned} \log \psi(x;m) &= \lim_{r \rightarrow \infty} r \log \left( \frac{2}{\pi} \arctan \frac{rx}{m} \right) = \lim_{\mu \rightarrow 0} \frac{\log \left( \frac{2}{\pi} \arctan \frac{x}{\mu m} \right)}{\mu} \\ &= \lim_{\mu \rightarrow 0} \frac{\frac{d}{d\mu} \log \left( \frac{2}{\pi} \arctan \frac{x}{\mu m} \right)}{\frac{d}{d\mu} \mu} = \lim_{\mu \rightarrow 0} \frac{\left( \frac{2}{\pi} \right) \left\{ \frac{1}{1 + \left( \frac{x}{\mu m} \right)^2} \right\} \left( \frac{-x}{m \mu^2} \right)}{\frac{2}{\pi} \arctan \frac{x}{\mu m}}. \end{aligned}$$

The denominator of this expression goes to unity, so

$$\begin{aligned} \log \psi(x;m) &= \lim_{\mu \rightarrow 0} \frac{-2x}{\pi m \left[ \mu^2 + \left( \frac{x}{m} \right)^2 \right]} \\ &= \frac{-2m}{\pi x}, \end{aligned}$$

Hence

$$\psi(x;m) = e^{-\left(\frac{2m}{\pi x}\right)}. \quad (6)$$

From equation (6) we see that the fractiles of the largest discontinuity in an interval are proportional to the standard scale of the total Cauchy increment. Infact, the median largest discontinuity is 0.918m, so that the median largest discontinuity is actually smaller than m, the median absolute total Cauchy increment over the interval. The fractiles of the  $i^{\text{th}}$  largest discontinuity fall toward Zero as  $i$  becomes large. We again divide the entire interval into  $r$  subintervals and consider the distribution of the  $k^{\text{th}}$  largest increment. The probability that some  $k-1$  of the  $r$  increments will be larger than  $\xi$  is  $[1 - H(\xi; m/r)]^{k-1}$ . The probability that all the others will be smaller than  $\xi$  is  $H(\xi; m/r)^{r-k+1}$ . The probability that the largest of these others is between  $\xi$  and  $\xi + d\xi$  is  $dH^{r-k+1} = (r - k + 1)H^{r-k} h d\xi$ . There are  $\binom{r}{k-1}$  ways in which we can select the  $k-1$  intervals in which  $k-1$  largest increments occur. Therefore the probability that the  $k^{\text{th}}$  largest increment is less than  $x$  is  $\binom{r}{k-1} \int_0^x (1-H)^{k-1} dH^{r-k+1}$ , and  $\psi^{(K)}(x; m)$ , the probability that the  $k^{\text{th}}$  largest discontinuity is less than  $x$ , is

$$\psi^{(K)}(x; m) = \lim_{r \rightarrow \infty} \binom{r}{k-1} \int_0^x (1-H)^{k-1} dH^{r-k+1} \tag{7}$$

Now consider the difference

$$\delta_{k+1} = \psi^{(K+1)} - \psi^{(K)} \tag{8}$$

$$= \lim_{r \rightarrow \infty} \left[ \binom{r}{k} \int_0^x (1-H)^k dH^{r-k} - \binom{r}{k-1} \int_0^x (1-H)^{k-1} dH^{r-k+1} \right]. \tag{9}$$

Then performing integration by parts on the first integral in (9) with  $v=(1-H)^k$  and  $d\mu=dH^{r-k}$ , We have

$$\begin{aligned} \delta_{k+1} &= \lim_{r \rightarrow \infty} \left\{ \frac{r^{(k)}}{k!} \left[ (1-H)^k H^{r-k} - \int_0^x (1-H)^{k-1} (-k) H^{r-k} h d\xi \right] - \frac{r^{(k-1)}}{(k-1)!} \int_0^x (1-H)^{k-1} (r-k+1) H^{r-k} h d\xi \right\} \\ &= \frac{1}{k!} \lim_{r \rightarrow \infty} \left\{ r^{(k)} (1-H)^k H^{r-k} + [k r^{(k)} - k r^{(k-1)} (r-k+1)] \int_0^x (1-H)^{k-1} H^{r-k} h d\xi \right\} \\ &= \frac{1}{k!} \lim_{r \rightarrow \infty} [r^{(k)} (1-H)^k H^{r-k} + 0], \end{aligned} \tag{10}$$

Since  $(r-k+1)r^{(k-1)} = r^{(k)}$ . Now from (5) and (6),

$$\lim_{r \rightarrow \infty} H^{r-k} = \lim_{r \rightarrow \infty} H^r = e^{-\left(\frac{2m}{\pi x}\right)}. \tag{11}$$

### III. EXAMPLE

The Fig. (1) shows that there was an effect of time ( $p < 0.001$ ), but no effect of treatment and no treatment by time interaction, for plasma GIP concentrations. Glucose was infused at 3.5kcal/min into the duodenum of eight healthy males (age 18-59 yrs) over 60 minutes on the first day into an isolated 60-cm segment of the proximal small intestine (short segment infusion). on the second day the same amount of glucose was infused with access to the entire small intestine (long-segment infusion). During both conditions, plasma GIP increased rapidly from baseline at  $t=10$  min ( $p < 0.001$ ) and plateaued at 30 min. There was no effect of treatment on the areas under curves for plasma GIP [12].

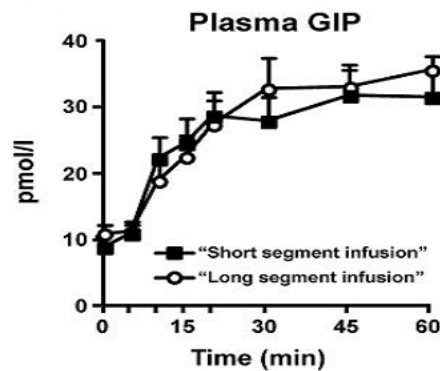


Fig. (1)

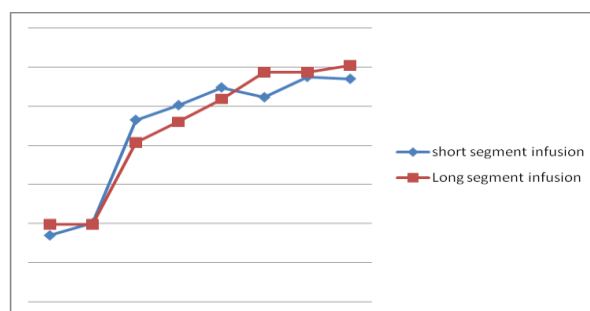


Fig. (2)

#### IV. CONCLUSION

Evaluation of the GIP response after infusing the glucose into the duodenum of eight healthy males over 60 min on the first day into an isolated 60-cm segment of proximal small intestine and on the second day the same amount of glucose was infused to entire small intestine which is fitted with the fractiles of the largest discontinuity in an interval are proportional to the standard scale of the total Cauchy distribution is graphically shown in Fig(2). The result coincides with the mathematical and medical report.

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